SOLUTIONS TO EXPONENT LAWS PROBLEMS

PRODUCT OF POWERS $a^m a^n = a^{m+n}$

Question ① $(x^3y)(x^4y^5) = x^3y^1 \cdot x^4y^5 = x^3 \cdot x^4 \cdot y^1y^5$

$= x^{3+4}y^{1+5} = \boxed{x^7y^6}$

Question ② $(x^2y)(x^3y^4) = x^2y^1 \cdot x^3y^4 = x^2 \cdot x^3 y^1y^4$

$= x^{2+3}y^{1+4} = \boxed{x^5y^5}$

Question ③ $(x^8y)(x^{31}y^{42}) = x^8 \cdot y^1 \cdot x^{31} \cdot y^{42} = x^8 \cdot x^{31} y^1 \cdot y^{42}$

$= x^{8+31} \cdot y^{1+42} = \boxed{x^{39}y^{43}}$

Question ④ $(x^7y)(x^{12}y^{40}) = x^7y^1 \cdot x^{12}y^{40} = x^7 \cdot x^{12} \cdot y^1y^{40}$

$= x^{7+12}y^{1+40} = \boxed{x^{19}y^{41}}$

Question ⑤ $(x^7y^6)(x^5y^5)(x^{39}y^{43})(x^{19}y^{41}) =$

All answers product

$x^7x^5x^{39}x^{19}\, y^6y^5y^{43}y^{41} = x^{7+5+39+19}\, y^{6+5+43+41}$

$= \boxed{x^{70}y^{95}}$

POWER OF POWER $(a^m)^n = a^{mn}$

QUESTION ① $(x^2y)^3 = (x^2y^1)^3 = (x^2y^1)(x^2y^1)(x^2y^1)$

$= x^{2\cdot3}y^{1\cdot3} = \boxed{x^6y^3}$

Question ② $(x^4y)^3 = (x^4y^1)^3 = (x^4y^1)(x^4y^1)(x^4y^1)$

$= x^{4\cdot3}y^{1\cdot3} = \boxed{x^{12}y^3}$

Question ③ $(x^8y)^5 = (x^8y^1)^5 = (x^8y^1)(x^8y^1)(x^8y^1)(x^8y^1)(x^8y^1)$

$= x^{8\cdot5}y^{1\cdot5} = \boxed{x^{40}y^5}$

Question ④ $(x^4y)^9 = (x^4y^1)^9 = (x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)(x^4y^1)$

$= x^{4\cdot9}y^{1\cdot9} = \boxed{x^{36}y^9}$

Question ⑤ $(x^6y^3)(x^{12}y^3)(x^{40}y^5)(x^{36}y^9) =$

all answers product

$x^6x^{12}x^{40}x^{36}y^3y^3y^5y^9 = x^{6+12+40+36}y^{3+3+5+9}$

$= \boxed{x^{94}y^{20}}$

POWER OF A PRODUCT $(a^n b^m)^T = a^{nT} b^{mT}$

Question ① $(2x^{-4}y)^{-3} = (2^1 x^{-4} y^1)^{-3} = 2^{(1)(-3)} x^{(-4)(-3)} y^{(1)(-3)}$

$= 2^{-3} x^{12} y^{-3} = \frac{1}{2^3} \cdot \frac{x^{12}}{1} \cdot \frac{1}{y^3} = \boxed{\frac{x^{12}}{8 \cdot y^3}}$

Question ② $(4x^{-4}y)^2 = (4^1 x^{-4} y^1)^2 = 4^{1(2)} \cdot x^{(-4)(2)} \cdot y^{(1)(2)}$

$= 4^2 x^{-8} y^2 = \frac{16}{1} \cdot \frac{1}{x^8} \cdot \frac{y^2}{1} = \boxed{\frac{16y^2}{x^8}}$

Question ③ $(3x^{-4}y)^3 = (3^1 x^{-4} y^1)^3 = 3^{1(3)} \cdot x^{(-4)(3)} \cdot y^{(1)(3)}$

$= 3^3 \cdot x^{-12} y^3 = \frac{27}{1} \cdot \frac{1}{x^{12}} \cdot \frac{y^3}{1} = \boxed{\frac{27y^3}{x^{12}}}$

Question ④ $(8x^2y)^{-2} = (8^1 x^2 y^1)^{-2} = 8^{1(-2)} \cdot x^{2(-2)} \cdot y^{1(-2)}$

$= 8^{-2} x^{-4} y^{-2} = \frac{1}{8^2} \cdot \frac{1}{x^4} \cdot \frac{1}{y^2} = \boxed{\frac{1}{64x^4y^2}}$

Question ⑤ all products from answers

$\left(\frac{x^{12}}{8y^3}\right)\left(\frac{16y^2}{x^8}\right)\left(\frac{27y^3}{x^{12}}\right)\left(\frac{1}{64x^4y^2}\right) = \frac{16 \cdot 27}{8 \cdot 64} \cdot \frac{x^{12}}{x^8 x^{12} x^4} \cdot \frac{y^2 y^3}{y^3 \cdot y^2}$

$= \frac{432}{512} \cdot \frac{x^{12}}{x^{8+12+4}} \cdot \frac{y^{2+3}}{y^{3+2}} = \frac{27}{32} \cdot \frac{x^{12}}{x^{24}} \cdot \frac{y^5}{y^5} = \boxed{\frac{27}{32x^{12}}}$

SOLUTIONS negative exponent rule $a^{-m} = \frac{1}{a^m}$

Question ① $(x^{-2}y)^{-3} = (x^{-2}y^1)^{-3} = x^{(-2)(-3)}y^{1\cdot(-3)}$

$= x^6 y^{-3} = \frac{x^6}{1}\cdot\frac{1}{y^3} = \boxed{\frac{x^6}{y^3}}$

Question ② $(x^2y)^{-4} = (x^2y^1)^{-4} = x^{2(-4)}y^{1(-4)} = x^{-8}y^{-4}$

$= \frac{1}{x^8}\cdot\frac{1}{y^4} = \boxed{\frac{1}{x^8y^4}}$

Question ③ $(x^{-5}y)^2 = (x^{-5}y^1)^2 = x^{(-5)(2)}y^{1(2)} \quad x^{-10}y^2$

$= \frac{1}{x^{10}}\frac{y^2}{1} = \boxed{\frac{y^2}{x^{10}}}$

Question ④ $(x^8y)^{-4} = (x^8y^1)^{-4} = x^{8(-4)}y^{1(-4)} = x^{-32}y^{-4}$

$= \frac{1}{x^{32}}\cdot\frac{1}{y^4} = \boxed{\frac{1}{x^{32}y^4}}$

Question ⑤
all answers
product

$\frac{x^6}{y^3}\cdot\frac{1}{x^8y^4}\cdot\frac{y^2}{x^{10}}\cdot\frac{1}{x^{32}y^4} = \frac{x^6}{x^8x^{10}x^{32}}\cdot\frac{y^2}{y^3\cdot y^4\cdot y^4} = \frac{x^6y^2}{x^{50}y^{11}}$

$= x^{6-50}y^{2-11} = x^{-44}y^{-9} = \boxed{\frac{1}{x^{44}y^9}}$

Power of a Quotient $\left(\frac{a^m}{b^n}\right)^r = \frac{a^{mr}}{b^{nr}}$

Question ① $\left(\frac{2x^{-4}y}{5x}\right)^{-3} = \left[\frac{2^1 x^{-4} y^1}{5^1 x^1}\right]^{-3} = \frac{2^{-3} x^{12} y^{-3}}{5^{-3} x^{-3}} = \frac{5^3}{2^3} \frac{x^{12} x^3}{1} \cdot \frac{1}{y^3}$

$= \frac{125}{8} \cdot \frac{x^{12+3}}{1} \cdot \frac{1}{y^3} = \boxed{\frac{125x^{15}}{8y^3}}$

Question ② $\left(\frac{2x^2y}{5x^{-4}}\right)^2 = \left(\frac{2^1 x^2 y^1}{5^1 x^{-4}}\right)^2 = \frac{2^2 x^4 y^2}{5^2 x^{-8}} = \frac{4}{25} \frac{x^4 \cdot x^8}{1} \cdot \frac{y^2}{1}$

$= \frac{4}{25} \cdot \frac{x^{4+8}}{1} \cdot \frac{y^2}{1} = \boxed{\frac{4x^{12}y^2}{25}}$

Question ③ $\left(\frac{2x^3y}{5x^{-2}}\right)^{-3} = \left(\frac{2^1 x^3 y^1}{5^1 x^{-2}}\right)^{-3} = \frac{2^{-3} x^{-9} y^{-3}}{5^{-3} x^6} = \frac{5^3}{2^3} \cdot \frac{1}{x^9 x^6} \cdot \frac{1}{y^3}$

$= \frac{125}{8} \cdot \frac{1}{x^{9+6}} \cdot \frac{1}{y^3} = \boxed{\frac{125}{8x^{15}y^3}}$

Question (4) Power of Quotient

$$\left(\frac{2x^3y}{5x^{-6}}\right)^2 = \left(\frac{2^1x^3y^1}{5^1x^{-6}}\right)^2 = \frac{2^2x^6y^2}{5^2x^{-12}} = \frac{4}{25}\cdot\frac{x^6x^{12}}{1}\cdot\frac{y^2}{1}$$

$$= \frac{4}{25}\cdot\frac{x^{6+12}}{1}\cdot\frac{y^2}{1} = \boxed{\frac{4x^{18}y^2}{25}}$$

Powers that involve fractions / Perfect Powers

Note

x	1	2	3	4	5
x^2	1	$2^2=4$	$3^2=9$	$4^2=16$	$5^2=25$
x^3	1	$2^3=8$	$3^3=27$	$4^3=64$	$5^3=125$
x^4	1	$2^4=16$	$3^4=81$	$4^4=256$	$5^4=625$
x^5	1	$2^5=32$	$3^5=243$	$4^5=1024$	$5^5=3125$

Rational Exponents $x^{m/n} = \sqrt[n]{x^m}$

Note $x^{m/m} = x^1$ note; $\sqrt[m]{x^m} = x^{m/m} = x^1$

Question ① $(16x^{12})^{3/4} = (16^1 x^{12})^{3/4} = 16^{3/4} \cdot x^{12(3/4)}$

$= 16^{3/4} \cdot x^{36/4} = 16^{3/4} \cdot x^9$

Now $16^{3/4} = \sqrt[4]{16^3} = \sqrt[4]{(2^4)^3} = \sqrt[4]{2^{12}} = 2^{\frac{12}{4}} = \boxed{2^3 = 8}$

So $(16x^{12})^{3/4} = \sqrt[4]{16^3 x^{12(3)}} = \sqrt[4]{2^{12} x^{36}} = 2^{\frac{12}{4}} \cdot x^{\frac{36}{4}}$

$= 2^3 \cdot x^9 = \boxed{8x^9}$

Question ② $(81x^{24})^{3/4} = (3^4 x^{24})^{3/4} = 3^{4(3/4)} \cdot x^{24(3/4)}$

$= 3^{\frac{12}{4}} \cdot x^{72/4} = 3^3 \cdot x^{18} = \boxed{27x^{18}}$

Rational Exponents

Question (3) $(81x^{12})^{\frac{3}{2}} = (3^4 \cdot x^{12})^{3/2} = 3^{4(\frac{3}{2})} \cdot x^{12(\frac{3}{2})}$

$= 3^{\frac{12}{2}} \cdot x^{\frac{36}{2}} = \boxed{3^6 y^{18}} = \boxed{729 y^{18}}$

Question (4) $(25x^{12})^{-3/2} = (25^1 x^{12})^{-\frac{3}{2}} = (5^2 x^{12})^{-3/2}$

$= 5^{2(-3/2)} x^{12(-3/2)} = 5^{-6/2} \cdot x^{-\frac{36}{2}}$

$= 5^{-3} x^{-18} = \frac{1}{5^3} \frac{1}{x^{18}} = \boxed{\frac{1}{125x^{18}}}$

Question (5)

Now to Multiply $(8x^9)(27x^{18})(729x^{18})\left(\frac{1}{125x^{18}}\right)$

$= \frac{8 \cdot 27 \cdot 729}{125} \cdot \frac{x^9 x^{18} x^{18}}{x^{18}} = \frac{157464}{125} \cdot \frac{x^{9+18+18}}{x^{18}}$

used calculator

$= \frac{157464}{125} \cdot \frac{x^{45}}{x^{18}} = \frac{157464}{125} \cdot \frac{x^{45-18}}{1} = \boxed{\frac{157464\, x^{27}}{125}}$